4-7 Videos Guide

4-7a

- Rectangular-cylindrical conversions
  - $x = r\cos\theta$
  - $y = r\sin\theta$
  - $z = z$
  - $x^2 + y^2 = r^2$
  - $\tan\theta = \frac{y}{x}$

- The triple integral in cylindrical coordinates
  - $\iiint_E f(x, y, z)\, dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta)\, r\, dz\, dr\, d\theta$
  - Note that $dV = r\, dz\, dr\, d\theta$

Exercises:

- Sketch the solid described by the given inequalities.
  $0 \le \theta \le \pi/2, r \le z \le 2$
- Use cylindrical coordinates to evaluate $\iiint_E z\, dV$, where E is enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

4-7b

- Use cylindrical coordinates to evaluate $\iiint_E (x - y)\, dV$, where E is the solid that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$, above the xy-plane, and below the plane $z = y + 4$.

4.7c

- Use cylindrical coordinates to find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.
- Evaluate the integral by changing to cylindrical coordinates.

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \sqrt{x^2 + y^2}\, dz\, dy\, dx$$

4.7d

- Use cylindrical coordinates to find the volume of the solid that lies between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$.